Online Conference on

“Analysis and Approximation of Variational and Hemivariational Inequalities”

April 10-11, 2021

Abstracts of the talks

The meeting is organized as an online event via Zoom.

Organizers:
Weimin Han, weimin-han@uiowa.edu
Stanislaw Migorski, stanislaw.migorski@uj.edu.pl
Mircea Sofonea, sofonea@univ-perp.fr

Discontinuous Virtual Element Method for Elliptic Variational and Hemivariational Inequalities

Krzysztof Bartosz

Jagiellonian University
Faculty of Mathematics and Computer Science
Kraków, Poland

krzysztof.bartosz@uj.edu.pl

Abstract: In the last years the numerical methods for solving variational and hemivariational inequalities have been rapidly developed. Most of the results in this field are based on Galerkin approximation and use Finite Element Method (FEM), in particular. Recently, a generalisation of classical FEM has evolved in several directions. One of them is Discontinuous Galerkin Method (DGM), which very quickly became a new branch of numerical analysis. The second generalisation of FEM is Virtual Element Method (VEM), which has attracted attention of many scientists since it was described first in 2013. We try to join the both ideas and consider Discontinuous Virtual Element Method for problems modelled by variational and hemivariational inequalities. This is joint work with Paweł Szafraniec.

Fractional Derivatives with Respect to Time for Non-classical Heat Problem

Mahdi Boukrouche

CNRS UMR 5208, ICJ, Lyon University UJM, France

mahdi.boukrouche@univ-st-etienne.fr

Abstract: We consider the non-classical heat equation with Caputo fractional derivative with respect to the time variable in a bounded domain $\Omega \subset \mathbb{R}^+ \times \mathbb{R}^{d-1}$ for which the energy supply depends on the heat flux on a part of the boundary $S = \{0\} \times \mathbb{R}^{d-1}$ with homogeneous Dirichlet boundary condition on S, the periodicity on the other parts of the boundary and an initial condition. The problem is motivated by the modeling of the temperature regulation in the medium. The existence of solution to the considered problem is based on a Volterra integral of second kind in the time variable t with a parameter in $\mathbb{R}^{d-1}$, its solution is the heat flux $(y, \tau) \mapsto V(y, t) = u_x(0, y, t)$ on S, which is also an additional unknown of the considered problem. We establish that a unique local solution, exists and can be extended globally in time.

Efficient Numerical Methods and their Applications to the Complex Flow Field with Boundary Slip

Feifei Jing
Northwestern Polytechnical University
127 West Youyi Road, Xi'an, Shaanxi, 710072, P.R. China
ffjing@nwpu.edu.cn

Abstract: The boundary condition of fluid-flow is one of the most important factors to determine its hydrodynamic behaviors. In microfluidic systems, boundary slip may have a significant effect on the performance of such system. As an effective technique to catch the boundary slip phenomenon, numerical method provides some theoretical guidance for related experimental research. In this talk, a class of mathematical models with a nonlinear slip boundary condition of friction type is studied, which is used to describe the flow in the blood vessel of arteriosclerosis, as well as the possible slip phenomena. Due to the subdifferentiability of such boundary condition, these models can be characterized by variational or hemivariational inequalities. We will design stable and efficient numerical schemes, and establish priori error analyses for these variational inequalities. Numerical tests are reported to verify the theoretical results.

A Revisit of Elliptic Variational-Hemivariational Inequalities

Weimin Han
Department of Mathematics, University of Iowa
Iowa City, IA 52242-1410, USA
weimin-han@uiowa.edu

Abstract: An alternative approach is provided to establish the solution existence and uniqueness for elliptic variational-hemivariational inequalities. The new approach is based on elementary results from functional analysis (convex optimization and Banach fixed-point theorem), and thus removes the need of the notion of pseudomonotonicity and the dependence on abstract surjectivity results for pseudomonotone operators that are commonly used in the current literature on variational-hemivariational inequalities. This makes the theory of elliptic variational-hemivariational inequalities more accessible to applied mathematicians and engineers. Of independent interest, in the new approach, equivalent minimization principles are established for certain elliptic variational-hemivariational inequalities. Representative examples from contact mechanics are discussed to illustrate application of the theoretical results.

A Survey of Numerical Methods for Hemivariational Inequalities with Applications to Contact Mechanics

Michal Jureczka

Jagiellonian University
Faculty of Mathematics and Computer Science
Kraków, Poland

michal.jureczka@uj.edu.pl

Abstract: The main idea of this presentation is to describe implementations of three established methods for solving contact mechanic problems: direct optimization method, augmented Lagrangian method and primal-dual active set strategy. To do that, we first provide a brief study of an abstract nonsmooth optimization problem and prove existence of its unique solution. We later introduce a discrete numerical scheme to approximate this solution. Presented theory is applied to a sample static contact problem describing an elastic body in frictional contact with a foundation. Results obtained by listed algorithms on introduced contact problem are provided and reviewed. This presentation is result of a joint work with P. Bartman and A. Ochal.

Optimal Control of an Evolution Hemivariational Inequality Involving History-dependent Operators

Zhenhai Liu

Guangxi University for Nationalities
Guangxi, Nanning, P.R. China

zhhliu@hotmail.com

Abstract: In this talk, we introduce a class of feedback control systems described by an evolution hemivariational inequality involving history-dependent operators. Under the mild conditions, first, we show a priori estimates of the solutions to the feedback control system. Then, an existence theorem for the feedback control system is obtained by using the well-known Bohnenblust-Karlin fixed point theorem. Moreover, we study an optimal control problem driven by the feedback control system, and establish its solvability. Finally, a parabolic partial differential system with Clarke subgradient term is considered to illustrate the applicability of the theoretical results.

On a Class of Mixed Variational-Hemivariational Problems

Andaluzia Matei

Department of Mathematics, University of Craiova, Romania

andaluzia.matei@edu.ucv.ro

Abstract: We focus on a class of mixed variational-hemivariational problems,

$$\begin{aligned} a(u, v-u) + b(v-u, \lambda) + J^0(Tu; Tv - Tu) &\geq (f, v-u)_X \quad \text{for all} \quad v \in X, \\ b(u, \mu - \lambda) &\leq 0 \quad \text{for all} \quad \lambda \in \Lambda, \end{aligned}$$

looking for a pair solution $(u, \lambda) \in X \times \Lambda$ in a Hilbert spaces framework. Interesting generalizations and applications will be highlighted.

A Class of Elliptic Quasi-Variational-Hemivariational Inequalities with Applications to Fluid Flow Models

Stanisław Migórski

Jagiellonian University
Faculty of Mathematics and Computer Science
Kraków, Poland

stanislaw.migorski@uj.edu.pl

Abstract: In this paper we study a new class of quasi–variational–hemivariational inequalities in reflexive Banach spaces. The inequalities contain a convex potential, a locally Lipschitz superpotential, and a implicit obstacle set of constraints. Solution existence and compactness of the solution set to the inequality problem are established based on the Kakutani–Ky Fan fixed point theorem. The applicability of the results is illustrated by the steady-state Oseen model of a generalized Newtonian incompressible fluid with mixed boundary conditions. The latter involve a unilateral boundary condition, the Navier slip condition, a nonmonotone version of the nonlinear Navier-Fujita slip condition, and the threshold slip and leak condition of frictional type.

Shape Optimization of Elasto-Plastic Frictional Contact Problems

Andrzej Myśliński

Systems Research Institute
Polish Academy of Sciences, Warsaw, Poland
myslinsk@ibspan.waw.pl

Abstract: A shape optimization problem is considered for the variational inequality governing the contact between elasto-plastic rather than elastic structures and the foundation. Assuming the small strain plasticity model this contact phenomenon with a given friction is described by the system of the coupled variational inequalities in terms of the displacement and the generalized stress of the structure. The domain occupied by the structure is the control variable. The structural optimization problem consists in finding such material distribution inside the domain occupied by the structure in contact to minimize the normal contact stress. The system of variational inequalities governing the contact problem is approximated by the system of nonlinear equations depending on the regularization parameter using the penalization and smoothing techniques. The existence of solutions to the state system is shown. The differentiability of the control-to-state map is proved. The shape differentiability of the cost functional is shown. First-order necessary optimality conditions are formulated. Generalized Newton as well as the level set method are used to solve numerically this structural optimization problem. Numerical results are provided.

Prox-regular and Subsmooth Sweeping Process

Florent Nacry

Laboratoire de Mathématiques et Physique
University of Perpignan Via Domitia
52 Avenue Paul Alduy, 66860 Perpignan, France
florent.nacry@univ-perp.fr

Abstract: Sweeping process has been introduced and studied by J.J. Moreau in 1971 as a constrained first order differential inclusion. Such evolution problems are described through the normal cone of a (convex) moving set and can be seen as a particular (differential) variational inequality. Due to its many applications (crowd motion, nonregular electrical circuits, allocation mechanisms in Economics, nonsmooth mechanics, . . .), various and numerous extensions of the original problem by Moreau has been developed over the years (BV, nonconvex, second order, stochastic, in Banach spaces, perturbed, . . .).

Roughly speaking, the so-called Moreau sweeping process can be handled in three major ways that we will briefly recall in our presentation: regularization (via ODE), reduction (to an unconstrained differential inclusion) and catching-up algorithm (a kind of Euler's explicit scheme).

This presentation is devoted to the existence of a new evolution problem which encompasses two among the most popular variants of sweeping process. Here, our moving set will be prox-regular or subsmooth in a general Hilbert space and allowed to jump. After giving the necessary preliminaries on prox-regularity and subsmoothness, we will focus on the construction (thanks to a suitable new version of the Moreau's catching-up algorithm) of a trajectory solution.

A Class of Impulsive Evolution Inclusions with a History-dependent Operator

Anna Ochal

Jagiellonian University
Faculty of Mathematics and Computer Science
Kraków, Poland

anna.ochal@uj.edu.pl

Abstract: We consider nonlinear first order impulsive evolution inclusions with a history-dependent operator. We provide results on the existence of a solution to the Cauchy problem and the compactness of the solution set. An application to a time-dependent semipermeability problem with impulses and a history-dependent operator illustrates the abstract results. This is a joint work with S. Migórski (Krakow, Poland).

Almost History-dependent Variational-Hemivariational Inequality for Viscoelastic Frictional Contact Problem

Dariusz Pączka

Warsaw University of Technology, Warsaw, Poland

d.paczka@mini.pw.edu.pl

Abstract: The talk is devoted to the study of a quasistatic frictional contact problem for a viscoelastic material with a general constitutive law described by functions which have a non-polynomial growth. The contact condition is modeled by a subdifferential friction condition with memory and the normal compliance with limited penetration and memory. The weak formulation of the problem is an almost history-dependent variational-hemivariational inequality for the velocity field. We provide existence and uniqueness result on an abstract inclusion with an almost history-dependent operator and a variational-hemivariational inequality in the reflexive Orlicz–Sobolev space. Finally, we present a result on the existence of a unique weak solution to the contact problem. This is joint work with Stanisław Migórski.

An Implicit Sweeping Process in Contact Mechanics

Mircea Sofonea

Laboratoire de Mathématiques et Physique
University of Perpignan Via Domitia
52 Avenue Paul Alduy, 66860 Perpignan, France
sofonea@univ-perp.fr

Abstract: We consider a mathematical model which describes the quasistatic contact of a viscoelastic body with a deformable obstacle covered by a layer of viscous fluid. The variational formulation of the model is in the form of a sweeping process for the strain field. We use this formulation to obtain the existence of a unique weak solution to the problem as well as its continuous dependence with respect to the data. The proofs are based on arguments of sweeping process, convexity and history-dependent operators. Next, we consider a general associated optimal control problem for which we prove the existence of optimal pairs. We provide relevant examples which differ by the choice of the control and cost function, together with the corresponding mechanical interpretations.

Existence, Comparison, Monotonicity and Convergence Results for a Class of Elliptic Hemivariational Inequalities

Domingo A. Tarzia

Universidad Austral and CONICET, Rosario, Argentina
DTarzia@austral.edu.ar

Abstract: In this paper we study a class of elliptic boundary hemivariational inequalities which originates in the steady-state heat conduction problem with non-monotone multivalued subdifferential boundary condition on a portion of the boundary described by the Clarke generalized gradient of a locally Lipschitz function. First, we prove a new existence result for the inequality employing the theory of pseudomonotone operators. Next, we give a result on comparison and monotonicity of solutions, and provide sufficient conditions that guarantee the asymptotic behavior of solution, when the heat transfer coefficient tends to infinity. Further, we show a result on the continuous dependence of solution on the internal energy and heat flux. Finally, some examples of convex and nonconvex potentials illustrate our hypotheses.

This is a joint work with Claudia M. Gariboldi (Universidad Nacional de Río Cuarto, Río Cuarto, Argentina), and Stanisław Migórski and Anna Ochal (Jagiellonian University, Krakow, Poland).

Virtual Element Method for General Elliptic Hemivariational Inequalities

Fei Wang
Xi'an Jiaotong University, Xi'an, P.R. China
feiwang.xjtu@xjtu.edu.cn

Abstract: An abstract framework of the virtual element method is established for solving general elliptic hemivariational inequalities with or without constraint, and a unified a priori error analysis is given for both cases. Then, virtual element methods of arbitrary order are applied to solve three elliptic hemivariational inequalities arising in contact mechanics, and optimal order error estimates are shown with the linear virtual element solutions. Numerical simulation results are reported on several contact problems; in particular, the numerical convergence orders of the lowest order virtual element solutions are shown to be in good agreement with the theoretical predictions. This is joint work with Bangmin Wu and Weimin Han.

Convergence Results for Elliptic Variational-Hemivariational Inequalities

Yi-bin Xiao
University of Electronic Science and Technology of China
Chengdu, P.R. China
xiaoyb9999@hotmail.com

Abstract: In this talk, we consider an elliptic variational-hemivariational inequality $\mathcal{P}$ in a reflexive Banach space, governed by a set of constraints K, a nonlinear operator A, and an element f. We associate to this inequality a sequence $\{\mathcal{P}_n\}$ of variational-hemivariational inequalities such that, for each $n \in \mathbb{N}$, inequality $\mathcal{P}_n$ is obtained by perturbing the data K and A and, moreover, it contains an additional term governed by a small parameter ε_n. The unique solvability of $\mathcal{P}$ and, for each $n \in \mathbb{N}$, the solvability of its perturbed version $\mathcal{P}_n$, are guaranteed by an existence and uniqueness result obtained in literature. Denote by u the solution of Problem $\mathcal{P}$ and, for each $n \in \mathbb{N}$, let u_n be a solution of Problem $\mathcal{P}_n$. The main result of this paper states the strong convergence of $u_n \to u$ in X, as $n \to \infty$. We show that the main result extends a number of results previously obtained in the study of Problem $\mathcal{P}$. Finally, we illustrate the use of our abstract results in the study of a mathematical model which describes the contact of an elastic body with a rigid-deformable foundation and provide the corresponding mechanical interpretations.