Linear bound on the irregularity strength and the total irregularity strength of graphs

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Abstract

Let G be a simple graph of order n with no isolated edges and at most one isolated vertex. For a positive integer w, a w-weighting of G is a map $f : E(G) \to \{1, 2, \ldots, w\}$. An irregularity strength of G, $s(G)$, is the smallest w such that there is a w-weighting of G for which $\sum_{e:u\in e} f(e) \neq \sum_{e:v\in e} f(e)$ for all pairs of different vertices $u, v \in V(G)$. A tight result by Nierhoff says that $s(G) \leqslant n - 1$. We show a new general upper bound, which is linear in n/δ, hence better starting from a given δ upwards. In the case of the d-regular graphs, we obtain a better linear function of n/d as an upper bound on $s(G)$, which corresponds with the conjecture by Faudree and Lehel that $s(G) \leqslant n/d + c$ for some absolute constant c. The recently introduced total version of the problem is also discussed and supported by a number of new bounds, also linear in n/δ.

Keywords: irregularity strength, total irregularity strength, graph weighting, graph labelling
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