

MATEMATYKA  
DYSKRETNA

[www.ii.uj.edu.pl/preMD/](http://www.ii.uj.edu.pl/preMD/)

Rafał KALINOWSKI  
and Mariusz WOŹNIAK

*Edge-distinguishing  
index of a graph*

Preprint Nr MD 050  
(otrzymany dnia 23.12.2010)

Kraków  
2010

Redaktorami serii preprintów *Matematyka Dyskretna* są:  
Wit FORYŚ,  
prowadzący seminarium *Słowa, słowa, słowa...*  
w Instytucie Informatyki UJ  
oraz  
Mariusz WOŹNIAK,  
prowadzący seminarium *Matematyka Dyskretna - Teoria Grafów*  
na Wydziale Matematyki Stosowanej AGH.

# Edge-distinguishing index of a graph

Rafał Kalinowski and Mariusz Woźniak\*

Department of Applied Mathematics  
AGH University of Science and Technology  
Cracow, Poland  
E-mail:{kalinows, mwozniak}@agh.edu.pl

December 23, 2010

## Abstract

We introduce a concept of edge-distinguishing colourings of graphs. A closed neighbourhood of an edge  $e \in E(G)$  is a subgraph  $N[e]$  induced by the edge  $e$  and all the edges adjacent to it. We say that a colouring  $c : E(G) \rightarrow C$  distinguishes two edges  $e_1$  and  $e_2$  if there does not exist an isomorphism  $\varphi$  of  $N[e_1]$  onto  $N[e_2]$  such that  $\varphi(e_1) = e_2$ , and  $\varphi$  preserves colours of  $c$ . An edge-distinguishing index of a graph  $G$  is the minimum number  $\chi'_e(G)$  of colours in a proper colouring  $c : E(G) \rightarrow C$  which distinguishes every two distinct edges of  $G$ . Such a colouring is called edge-distinguishing. We determine edge-distinguishing index for cycles, paths and complete graphs.

*Keywords:* proper edge colouring, chromatic index, Eulerian tours in multigraphs

*AMS Subject Classifications:* 05C15

## 1 Introduction

We use standard graph theory notation and terminology. Let  $G$  be a simple graph with the vertex set  $V(G)$  and the edge set  $E(G)$ . As usually,  $\chi'(G)$

---

\*The research partially supported by the Polish Ministry of Science and Higher Education