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Preprint Nr MD 070
(otrzymany dnia 24.06.2013)

**Kraków
2013**

Endomorphism Breaking in Graphs

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19 January 2013

Abstract

We introduce the *endomorphism distinguishing number* $D_e(G)$ of a graph G as the least cardinal d such that G has a vertex coloring with d colors that is only preserved by the trivial endomorphism. This generalizes the notion of the distinguishing number $D(G)$ of a graph G, which is defined for automorphisms instead of endomorphisms.

As the number of endomorphisms can vastly exceed the number of automorphisms, the new concept opens challenging problems, several of which are presented here. In particular, we investigate relationships between $D_e(G)$ and the endomorphism motion of a graph. Moreover, we extend numerous results about the distinguishing number of finite and infinite graphs to the endomorphism distinguishing number. This is the main concern of the paper.

Key words: Distinguishing number; Endomorphisms; Infinite graphs;

AMS subject classification (2000): 05C25, 05C80, 03E10

*The research was supported by the Austrian Science Fund (FWF): project W1230, and by ARRS Slovenia within the EUROCORES Programme EUROGIGA/GReGAS of the European Science Foundation.

†The research was partially supported by the Polish Ministry of Science and Higher Education.

‡The research was supported by the Austrian Science Fund (FWF): project W1230.

§The research was partially supported by the Polish Ministry of Science and Higher Education.